Modeling Maintenance Productivity Measurement of Engineering Production Systems: Discrete Event Simulation Approach

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Abstract-- **Modeling and simulation of engineering production systems is one of the major concerns of productivity engineers for the establishment of productivity standards in virtually all functional areas of an organization. There are several activities in a metal fabrication system, such as an aluminium rolling mills, but the maintenance function is a major area, if optimised, can ensure its profitability. The maintenance functions have several parameters to monitor and measure and there is the need for an integrated approach to optimize the basket of parameters measured.**

This paper is a study of the maintenance fabrication system with a view at identifying suitable approaches in integrated and systematic maintenance productivity measurement and creating models for optimising total productivity. Discussing several approaches of maintenance productivity measurement it shows how understanding the impact of plant failure and repair/service distributions assists in enhancing maintenance productivity using discrete event system simulation.

Index Term-- **Optimisation, Productivity, Maintenance, Markov Chain, Queuing Theory, System Dynamics, Discrete Event Simulation, Composite**

I. INTRODUCTION

Modeling and simulation of engineering production systems and networks is one of the major concerns of productivity engineers for the establishment of productivity standards in virtually all functional areas of an organization. In metal fabrication system, defective end products are introduced during processing, fabrication or use of metals in service. Those that are produced early in the processing chain may be carried forward to later stages where they can cause processing problems with a consequent defective end product and low productivity.

The overall objective of the maintenance function should be to support the production department by keeping facilities in proper running condition at the lowest possible cost. In manufacturing circles the word productivity is used in a variety of sense some of which are conflicting or very qualitative, namely, "efficiency", "overall effectiveness", etc. Similarly, the definition of "productivity" is varied. Productivity is often confused with "output" or "profitability". Whilst a good total productivity implies profitability, the converse does not hold. Profitability is affected by market prices and accounting practice. Productivity is defined simply as a relationship of output to input. In sharp contrast to production, the performance of maintenance activity does not lend itself easily to expression in simple or unified figures. However, in the last two decades, the measurement of maintenance performance and productivity has engaged the attention of productivity engineers (Bolu, 2013).

Bolu, C. A (2013), attempts at identifying approaches in integrated and systematic maintenance productivity measurement and creating models for optimising total productivity in maintenance systems. He discusses visual yardstick, utility, queuing systems and simulations approaches for measurement of maintenance productivity and highlights markov chain approach for stochastic breakdowns in repairable systems.

(Priel, 1974) has written on maintenance organization particularly on performance ratios. He has identified twenty of such maintenance ratios. Some of the ratios are useful in establishing the basis for incentive scheme for maintenance personnel. (Hamlin, 1979) has shown various methods and (Alli, Ogunwolu, & Oke, 2011) applied same to measure maintenance productivity through their case studies. (Chan, Lau, Ip, Chan, & Kong, 2005), applying total productive maintenance approach to the electronics industry, (Eti, Ogaji, Probert, 2004) to the manufacturing industries in a developing country, (Lilly, Obiajulu, Ogaji, &Probert, 2007) to the petroleum-product marketing company and (Ahn, Abt, 2006), to the sawmills and planning mills industry provides examples of total productivity measurements in industry.

Another interesting point of view is provided by (Nanere, M, Fraser, I, Quazi, A, & D'Souza, C, 2007), who critically examines various methods for estimating productivity incorporating environmental effects and shows that adjusting for environmental impacts can result in higher and lower productivity depending on the assumed form of the damage. Although this was applies to the agriculture sector, this could be applicable to industrial environment, where work place hygiene and design could impact negatively or positively to productivity.

It can be seen that there are several ways of expressing maintenance productivity. The problem is how to model a working measurement scheme that gives good management information in areas critical to increasing productivity as well as being amenable to easy data collection.

(Priel, 1974) attempted to integrate the various measures. (Onwugbolu et al, 1988) and (Parida et al, 2009) extends the apparaoch. It starts with a number of performance measures as identified by (Priel, 1974) and then builds a composite number which is a weighted addition of the Utility values of all these performance measures. The approach can be stated as follows:

Let U_t be the utility value of the selected N performance measures, where $t = 1, 2 \ldots \ldots . N$
Let β_t be the scaling factors for the performance measures

Then the Maintenance Productivity is given by

$$Y_N = \sum_1^N \beta_t U_t$$

From the case study by (Alli et al , 2009), they selected the following six performance measures mentioned in (Priel, 1974):

$$Equipment\ Availability, U_1 = \frac{Downtime}{Downtime + Uptime}\ x\ 100\%$$

$$Emergency\ Failure\ Intensity, U_2 = \frac{Downtime}{Uptime}\ x\ 100\%$$

$$Cost\ of\ maintenance\ Hour, U_3 = \frac{Total\ Maintenance\ Cost}{Total\ Maintenance\ Hour}$$

$$Maintenance\ Cost\ Component, U_4 = \frac{Total\ Maintenance\ Cost}{Production\ Output}$$

$$Routine\ Service\ Workload, U_5 = \frac{Planned\ Maintenance\ Hour}{Total\ Maintenance\ Hour}$$

$$Cost\ Reduction\ Ratio, U_6 = \frac{Routine\ Service\ Workload}{Cost\ of\ Maintenance\ Hour}$$

The next step was to obtain the scaling factor, β_i, where $i = 1, 2, \ldots . 6$

The scaling factor is an index number obtained from the utility values from the plot of the performance measures over a period of years.

II.   METHODOLOGY

According to (Sterman, 2000) Du Pont organization looked at the result of an in-house benchmarking study documenting a large gap between Du Pont's maintenance record and those of the best-practice companies in the global chemical industry and they developed an interesting defect creation and elimination model. Prior to the modeling work maintenance was largely seen as a process of defect correction (repair of failed equipment) and the maintenance function was viewed as a cost to be minimized. It shifted the views to defect prevention and defect elimination. The model centred on the physics of breakdown rather than cost minimization mentality.
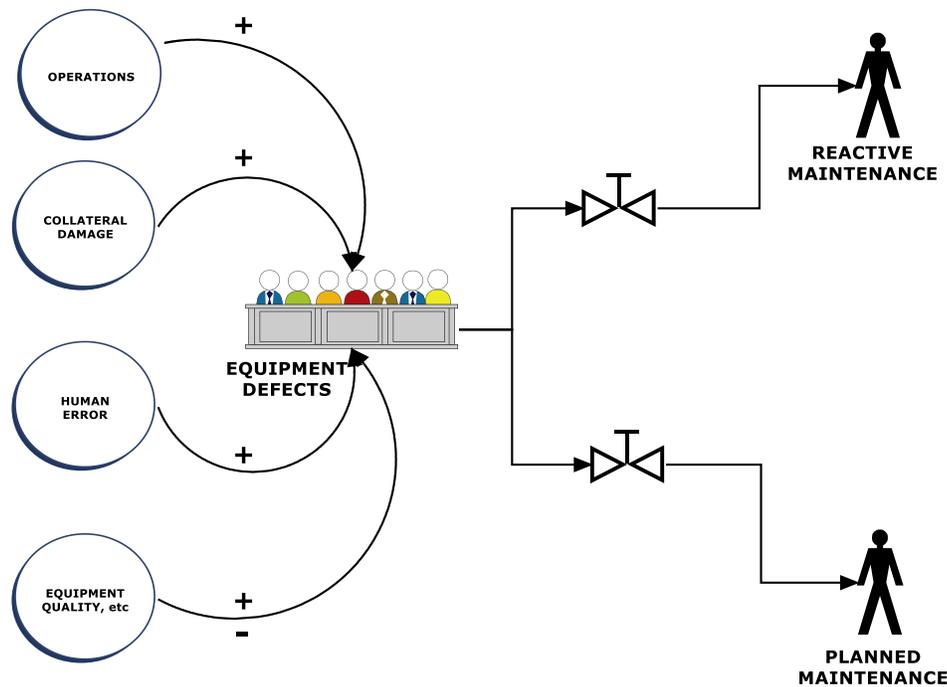
Fig. 1. System Dynamics Approach - Defects Accumulation (Bolu C. A, 2013)

The study postulates the following:

a. Equipment fails when a sufficient number of latent defects accumulate in it. Latent defects are any problem that might ultimately cause a failure. They include leaky oils seals in pumps, dirty equipment that causes bearing wear, pump and motor shaft that are out of alignment and cause vibration. The total number of latent defects in a plant's equipment is a stock (figure 1).

b. The stock of defects is drained by two flows: reactive maintenance (repair of failed equipment and planned maintenance (proactive repair of operable equipment}. As defects accumulate the chance of breakdown increase. Breakdown leads to more reactive maintenance, and, after repair, the equipment is returned to service and the stock of defects is reduced. Similarly, scheduled maintenance or equipment monitoring may reveal the presence of latent defects. The equipment is then taken out of service and the defects are corrected before breakdown occurs.

Obviously, since accumulation of defects are observable, this thinking can be modeled as a waiting line or queuing system and some of the solution methodologies of queuing theory could be useful in deriving the maintenance productivity, depending on the occurrence pattern (or arrival process) of the defects, the maintenance team service process and queue discipline of the maintenance policy.

The production of aluminium flat-rolled products can be divided into four major steps, namely hot strip production, strip rolling, thin-strip and foil rolling. The aluminium rolling mills maintenance was modeled as a queuing systems. To generate simulation data (Bolu, 2011) suggests:

a. Actual data could be used to calculate the desired statistics

b. Plot the histograms of the cumulative distribution of the breakdown times and the cumulative distribution of the service times and then generate sample breakdown and service times using these distributions.

c. Assume the actual data are values from certain theoretical distribution, and then sample from the theoretical distribution. To determine a theoretical distribution that would be a good approximation of the actual data, several possible distributions could be considered as candidates, and then the chi-square and/or Kolmogorov-Smirnov test could be used to determine the best distribution to use.

These above approach was used in generating simulation data for the breakdown data from a 4-hi Aluminium Rolling Mills collected over 3 years. Curve fitting was performed using MATLAB version 2011a Statistical Toolkit.

It can be classified with the following queuing model characteristics:

a. Defects arrival or breakdowns, λ, which is the distribution of the numbers of defects occurring, the number of defects that exceeds the threshold for

plant breakdown. It could also be the distribution of equipment breakdown

b. The service process, μ, which include the distribution of the time to eliminate or service a defect, the number of maintenance service team, and the arrangement of the maintenance service process (in parallel, series, etc).

c. Queuing discipline such as first come first served (FIFO), last in first served (LIFO), random selection, etc

The Erlang Maintenance Service Time: The *Erlang* distribution is a two-parameter family of distributions which is a special case of the more general *gamma* distribution. It permits more latitude in selecting a service-time distribution than the one-parameter exponential distribution. In fact, the exponential service time and constant service-time situations are special cases of the Erlang service time. In practical situations, the exponential distribution is unduly restrictive because it assumes that small service times are more probable than large service time, which is unusual for manufacturing plants. On the other hand, the Erlang distribution permits the flexibility of approximating almost any realistic service-time distribution.

We consider two-parameter Erlang distribution

$f(t; \mu, k) = \frac{(\mu k)^k t^{k-1} e^{-\mu k t}}{(k-1)!}$ $t > 0; \ \mu > 0; \ k = 1,2,3, \dots$

$E(T) = \frac{1}{\mu}$ for every $k = 1,2,3 \dots$

With $var(T) = \frac{1}{k\mu^2}$

For $k = 1$, the Erlang reduces to the exponential distribution

The following queuing statistics can be derived:

Average numbers of breakdowns	$L = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu - \lambda)}\right) + \frac{\lambda}{\mu}$	for $\lambda < \mu$ (1)
Average number of breakdowns in the queue	$L_q = \left(\frac{k+1}{2k}\right)\left(\frac{\lambda^2}{\mu(\mu - \lambda)}\right)$	for $\lambda < \mu$ (2)
Average time a breakdown stays in the system	$W = \frac{L}{\lambda}$	for $\lambda < \mu$ (3)
Average time a breakdown stays in the queue	$W_q = \frac{L_q}{\lambda}$	for $\lambda < \mu$ (4)

Weibull Distribution - Mean Time to Failure: The Weibull distribution can be considered as a generalization of the exponential distribution

$f(t) = \lambda\beta(\lambda t)^{\beta-1} e^{-(\lambda t)^\beta}$
$t > 0; \lambda, \beta >$
0 λ is the scale parameter and β the shape factor.
When $\beta = 1$ this yields the exponential distribution.

The following typical maintenance queuing systems were considered to study the productivity measurement of the production system:

Case 1: Single Breakdown Queue, Single-Server Maintenance Service Team
Case 2: Single Breakdown Queue and Multiple Maintenance Team
Case 3: Single Breakdown Queue and Multiple Maintenance Team
Case 4: Single Breakdown Queue and Multiple Maintenance Team in Series
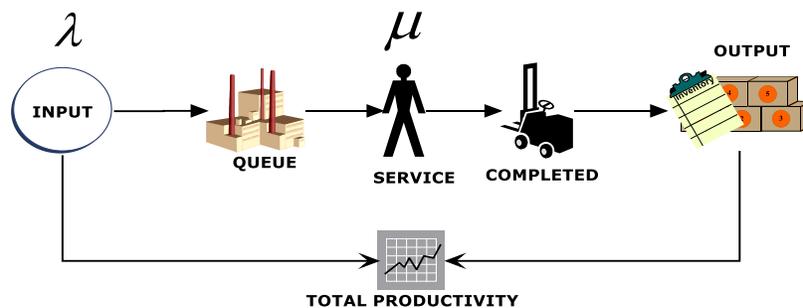Case 5: Multiple Breakdown Queue and Multiple Maintenance Team

Fig. 2. Single Breakdown Queue, Single-Server Maintenance Service Team

Single Breakdown Queue and Single Maintenance Team - Poisson Failure Rate and Exponential Maintenance Service Distribution [Infinite Queue – Infinite Source]

Let X = number of breakdowns or failures per week.

Then $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0,1,2 \dots;$ $\lambda > 0$ and mean $E(X) = \lambda$.
The parameter λ, then is the mean time to failure.

Also, let T = time to service a breakdown

Then $g(t) = \mu e^{-\mu t}$ $t > 0; \mu > 0$ and $E(T) = \frac{1}{\mu}$. the parameter, μ, mean service time.

From Queuing theory, the following queuing equations can be derived:

Average numbers of breakdowns
$$L = \frac{\lambda}{\mu - \lambda} \qquad (5)$$

Average number of breakdowns in the queue
$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} \qquad (6)$$

Average number of breakdowns in nonempty queues
$$L_w = \frac{\mu}{\mu - \lambda} \qquad (7)$$

Average time a breakdown stays in the system
$$W = \frac{1}{\mu - \lambda} \qquad (8)$$

Average time a breakdown stays in the queue
$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} \qquad (9)$$

Probability of more than k breakdowns in the system
$$P(n > k) = \left(\frac{\lambda}{\mu}\right)^{k+1} \qquad (10)$$

Probability of the time in the system is greater than t
$$P(T > t) = e^{-\mu\left(1 - \frac{\lambda}{\mu}\right)t} \qquad (11)$$

If any one of the quantities L, L_q, W or W_q can be determined, then others can be determined from the relationships:

$$L = \lambda W \qquad (12)$$

$$L_q = \lambda W_q \qquad (13)$$

$$W = W_q + \frac{1}{\mu} \qquad (14)$$

$$L = L_q + \frac{\lambda}{\mu} \qquad (15)$$

In practice, breakdowns queues cannot be infinite as this will definitely affect the total productivity of the production plant. When the capacity of the maintenance team is exceeded, service is procured from contract service team, of course, with the increased cost of maintenance and increased logistical effort.

Let M = breakdowns that can be accommodated by the in-house maintenance team. For the case, λ need not be less than the mean service time since the breakdown queue cannot build up without bound.

We have that

$$\mu_n = \mu \qquad for\ n = 1, 2, 3, \dots \dots$$

$$\lambda_n = \begin{cases} \lambda & for\ n = 0, 2, 3, \dots\dots M - 1 \\ 0 & for\ n = M,\ M + 1, \dots \end{cases}$$

We can derive the following queuing characteristics:

Fraction of time that there is no breakdown in the system
$$P_0 = \frac{1 - \lambda/\mu}{1 - \left(\frac{\lambda}{\mu}\right)^{M+1}} \qquad \begin{array}{l} for\ \lambda \neq \mu \\ (16) \end{array}$$

$$P_0 = \frac{1}{M + 1} \qquad \begin{array}{l} for\ \lambda = \mu \\ (17) \end{array}$$

Average numbers of breakdowns
$$L = \frac{(M + 1)\left(\frac{\lambda}{\mu}\right)^{M+1}}{1 - \left(\frac{\lambda}{\mu}\right)^{M+1}} \qquad \begin{array}{l} for\ \lambda \neq \mu \\ (16) \end{array}$$

$$L = M/2 \qquad \begin{array}{l} for\ \lambda = \mu \\ (17) \end{array}$$

Average number of breakdowns in the queue
$$L_q = L - (1 - P_0) \qquad (18)$$

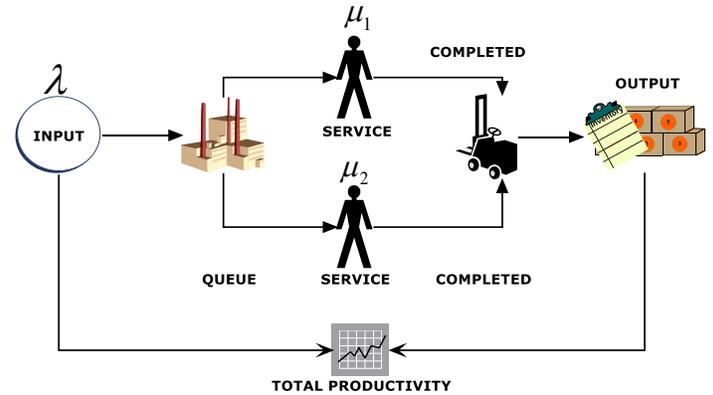## Case 2: Single Breakdown Queue and Multiple Maintenance Team

Fig. 3. Single Queue, Multiple Servers Model

We assume
  (a) s maintenance teams
  (b) Each maintenance team provides service at the same constant average rate μ
  (c) The average breakdown rate is constant, $\lambda_n = \lambda$ for all n
  (d) $\lambda < s\mu$

With these assumptions, the following queuing equations can be derived.

$$P_n = \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 \qquad n = 0, 1, \dots, s - 1 \qquad (19)$$

$$P_n = \frac{1}{s!\,s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0 \qquad n \geq s \tag{20}$$

$P(n \geq s)$ = probability a breakdown has to wait for servic

= probability of at least

s breakdown in the system

$= \sum_{n=s}^{\infty} P_n$

$$= \frac{\left(\frac{\lambda}{\mu}\right)^s P_0}{s!\,(1 - \frac{\lambda}{\mu s})} \tag{21}$$

$$P_0 = \frac{1}{\left\{\left[\sum_{n=0}^{s-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n\right] + \frac{1}{s!\,(1 - \frac{\lambda}{\mu s})}\left(\frac{\lambda}{\mu}\right)^s\right\}} \tag{22}$$

Average number of breakdowns in the queue	$L_q = \dfrac{\left(\frac{\lambda}{\mu}\right)^{s+1} P_0}{s.\,s!\,(1 - \frac{\lambda}{\mu s})^2}$	(23)
Average numbers of breakdowns	$L = L_q + \dfrac{\lambda}{\mu}$	(24)
Average time a breakdown stays in the system	$W = \dfrac{L}{\lambda}$	(25)
Average time a breakdown stays in the queue	$W_q = \dfrac{L_q}{\lambda}$	(26)

Probability of the time in the system is greater than t

$$P(T > t) = e^{-\mu t}\left\{1 + \frac{\left(\frac{\lambda}{\mu}\right)^s P_0\left[1 - e^{-\mu t(s - 1 - \frac{\lambda}{\mu})}\right]}{s!\,\left(1 - \frac{\lambda}{\mu s}\right)(s - 1 - \frac{\lambda}{\mu})}\right\} \tag{27}$$

Also in practice, breakdowns queues cannot be infinite as this will definitely affect the total productivity of the production plant. When the capacity of the maintenance team is exceeded, service is procured from contract service team, of course, with the increased cost of maintenance and increased logistical effort.

Let

$s = $ number of servers

$M = $ maximum number of breakdowns that can be accommodated b

$\lambda_n = \begin{cases} \lambda & for\ n = 0, 1, \dots M - 1 \\ 0 & for\ n = M, M+1, \dots \end{cases}$ Assume $1 < s < M$

$\mu_n = \begin{cases} n\mu & for\ n = 0, 1, \dots s \\ s\mu & for\ n = s+1, s+2, \dots \end{cases}$ Assume $1 < s < M$

Fraction of time that there is no breakdown in the system

$$P_0 = \frac{1}{\sum_{n=0}^{s}\left(\frac{1}{n!}\right)\left(\frac{\lambda}{\mu}\right)^n + \left(\frac{1}{s!}\right)\left(\frac{\lambda}{\mu}\right)^s \sum_{n=s+1}^{M}\left(\frac{\lambda}{\mu s}\right)^{n-s}} \tag{28}$$

Fraction of time that there is n breakdowns in the system

$$P_n = \begin{cases} \dfrac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0 & for\ n \leq s \\[2mm] \dfrac{1}{s!\,s^{n-s}}\left(\frac{\lambda}{\mu}\right)^n P_0 & for\ s < n \leq M \\[2mm] 0 & for\ n > M \end{cases} \tag{29}$$

Average number of breakdowns in the queue

$$L_q = \frac{P_0 \left(\frac{\lambda}{\mu}\right)^s \left(\frac{\lambda}{\mu s}\right)}{s!\,(1 - \frac{\lambda}{\mu s})^2}\left[1 - (\lambda/\mu s)^{M-s} - (M - s)\left(\frac{\lambda}{\mu s}\right)^{M-s}\left(1 - \frac{\lambda}{\mu s}\right)\right] \tag{30}$$

Average numbers of breakdowns

$$L = L_q + s - \sum_{n=0}^{s-1}(s - n)P_n \tag{31}$$

Case 3: Single Breakdown Queue and Multiple Maintenance Team

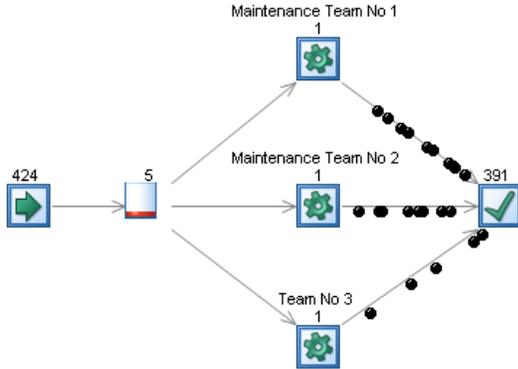**Case 4: Single Breakdown Queue and Multiple Maintenance Team in Series**

Fig. 4. SIMUL8 Implementation - Single Breakdown Queue and Multiple Maintenance Team
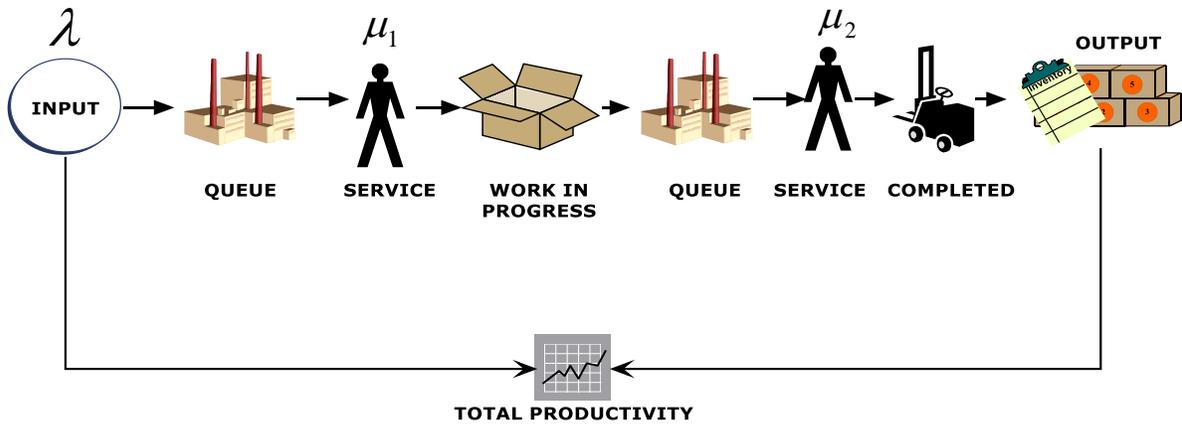

Fig. 5. Single Breakdown Queue, Multiple Maintenance Teams/Job Shops in Series

Fig. 6. SIMUL8 Implementation - Single Breakdown Queue, Multiple Maintenance Job Shops in Series

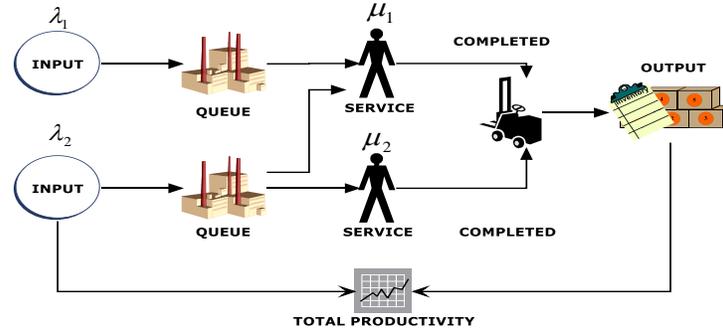**Case 5: Multiple Breakdown Queue and Multiple Maintenance Team**

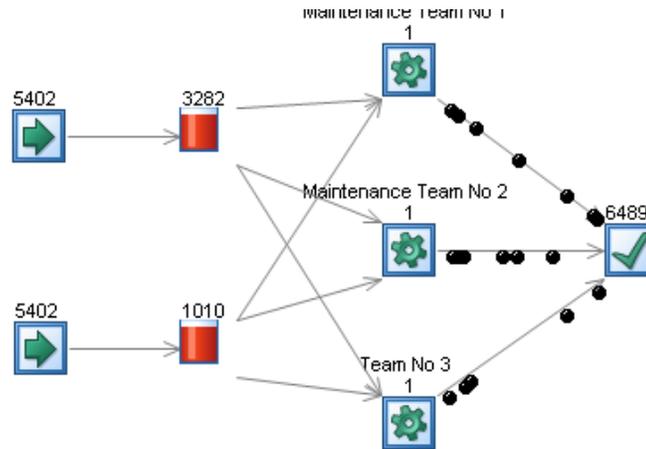Fig. 7. Multiple Queues, Multiple Maintenance Teams

Fig. 8. SIMUL8 Implementation - Multiple Breakdown Queue and Multiple Maintenance Team

III.   RESULTS

The results for the Single Breakdown Queue and Single Maintenance Team are presented both analytically and by the simulation using SIMUL8 Discrete Event simulation.

The following data was used:

Mean Time Between Failure	0.333 Days
Breakdown Distribution	Poisson
Mean Service Time	0.25 days
Mean Service Time Distribution	Exponential
Run Period	365 Days, One Financial Year
Working Hours	00:00a.m to 24:00 hrs, 7 Days

For productivity profile, the following data were used:

$$Productivity = \frac{Output}{Input}$$

(32)

$$for\ \lambda = 0.33333\ and\ \mu = 0.1, 0.2, \ldots \ldots \ldots \ldots .0.95$$

Optimal productivity profile can be obtained by varying $breakdown\ rate\ \lambda,\ and\ \text{s}ervice\ rate\ \mu$.

Fig. 9. SIMUL8 Implementation: Single Breakdown Queue, Single Maintenance Team
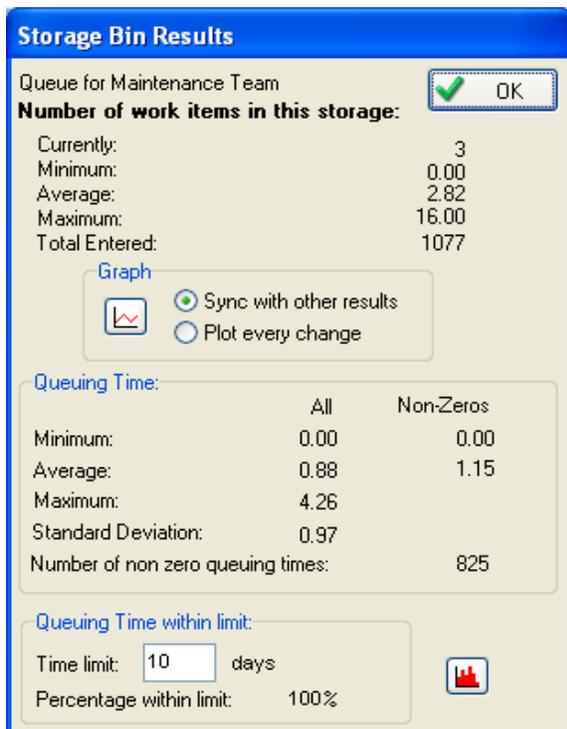
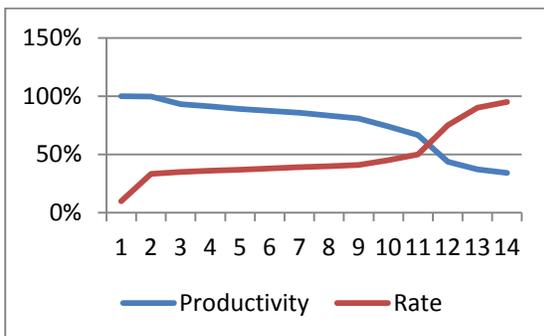Fig. 10. SIMUL8 Result: Single Breakdown Queue, Single Maintenance Team

Fig. 11. Maintenance Productivity: Single Breakdown Queue, Single Maintenance Team

IV.   DISCUSSIONS AND CONCLUSION

Comparisons of the several methodologies examined are summarized below:

a. Visual Representation is quite easy to determine but integration of the various parameters to a single value could be a challenge

b. Systems Dynamics approach is very subjective especially in data collection

c. The utility approach shares the same characteristics with the visual approach with the added subjectivity of determining the utility curve.

d. The queuing model seems to provide the platform to capture the characteristics of the engineering production system and providing given that the breakdown data parameters are statistically determined.

Method	Visual Presentation (Priel, 1974)	System Dynamics (Sterman, 2000)	Utility (Alli, et al, 2009)	Queuing Simulation (inclusive of environmental factor)
Ease of Determination	1	4	3	1
Integrated	3	2	3	1
Data Collection	1	4	3	3

1 = Excellent   2 = Good   3 = Fair   4 = Lot of work

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