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Digit And Iterative Digit Sum Of Fibonacci Numbers, Their Identities And Powers

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Abstract-
As the power of the Fibonacci numbers increases from 1 to the power of 5, the number of terms obtained from the digit sum decreases when compared with their respective positive integers. Digit sum may not be applicable to Fibonacci identities because of the heterogeneity of algebraic operations. Different recursive patterns were obtained for the different powers of the Fibonacci numbers considered. Prediction of the iterative digit sum of the powers of Fibonacci can be obtained by simple computation as illustrated in the paper. Iterative digit sum have added to the scientific literature as one of the various means of dividing integer sequences into distinct classes.

Keywords: Digit sum, iterative digit sum, Fibonacci numbers.

INTRODUCTION

Many researches have been done on the behavior, properties and characteristics of the Fibonacci numbers. This research is to give another dimension to the behavior of the digit and iterative digit sum of Fibonacci numbers, some selected identities, the squares, the cubes, the powers of 4 and 5. The study of the digit and iterative digit sum of number sequences is vital to give an insight on the behavior of large sets of numbers and to reduce computational complexity.

Digit sum of integer sequences has been studied for example, [1]. Researchers, for example, [2] argued that natural numbers can be represented as digit of Fibonacci numbers and the characteristics of natural numbers can be induced on the Fibonacci but [3] generalized it to the general case including digit sum. On the other hand, [4] studied the sum of digit of the expansion of Fibonacci numbers and related the results to the natural numbers. See [5] on details on the use of Mobius strip in the illustration of the effects of the bases of Fibonacci-like number sequences to the sum of digit.

Terr (1996) [6] proposed models for the prediction of the sum of digit of Fibonacci numbers in different digit but the models are not always successful. This work improved on the work of Terr, but in this case, on the aspect of iterative digit sum which gave a 99% prediction of iterative digit sum of any given Fibonacci numbers, their squares, cubes and higher powers.

The results are organized in the following order:

a). Digit and iterative sum of digit of Fibonacci numbers.
b). Digit and iterative sum of digit of some selected Fibonacci identities.
c). Digit and iterative sum of digit of squared Fibonacci Numbers.
d). Digit and iterative sum of digit of cubed Fibonacci Numbers.
e). Digit and iterative sum of digit of Fibonacci Numbers raised to the power of four.
f). Digit and iterative sum of digit of Fibonacci Numbers raised to the power of five.

Digit and iterative sum of digit of Fibonacci Numbers

The sequence of Fibonacci numbers (OEIS-A000045) is given by;

$$0, 1, 1, 2, 3, 5, 8, 13, 21, ... \qquad (A)$$

The sum of digit of Fibonacci numbers produced the sequence of positive integers with only few terms missing. Out of these numbers are some sequence of Palindromic numbers with the following sequence;

$$0,1,2,3,4,5,6,7,8,9,22,33,... \qquad (B)$$

The iterative digit sum of Fibonacci numbers produced all the single digit, the number 1 was the most frequent and 0, appeared only once. Based on the iterative digit sum, the set of the Fibonacci numbers can be subdivided into 9 distinct and non-overlapping classes. The reason is that the Fibonacci numbers modulo 9 is equivalent to the iterative digit sum of Fibonacci numbers modulo 9. For example;

$$F_{13} = 233 = 25*9+8 \text{ then, } 2+3+3 = 8 = 0*9+8$$

$$F_{15} = 610 = 67*9+7 \text{ then, } 6+1+0 = 7 = 0*9+7$$

$F_{16} = 987 = 109*9+6$ then, $9+8+7=24$

$= 2+4 = 6 = 0*9+6$

$F_{18} = 2584 = 287*9+1$ then, $2+5+8+4 = 19$

$= 1+9 = 10 = 1+0 = 0*9+1$

Fibonacci numbers can be divided based on modulo 9 of the iterative digit sum and the following 9 sequences can be obtained:

1, 1, 55, 2584, 28657, 75025, ... (C)

2, 10946, 196418, 1134903170, ... (D)

3, 21, 317811, 2178309, 32951280099, ... (E)

13, 1597, 1346269, 165580141, ... (F)

5, 4181, 514229, 433494437, ... (G)

987, 6765, 102334155, 701408733, ... (H)

34, 610, 3524578, 63245986, ... (I)

8, 89, 233, 377, 17711, 832040, 9227465, ... (J)

144, 46368, 14930352, 4807526976, ... (K)

The iterative digit sum of Fibonacci numbers produced numbers that are recursive, a pattern of numbers that are repeated after every 24th consecutive terms. This can also be used in predicting the iterative digit sum of large Fibonacci numbers. This is shown in **Table 1** and **Figure 1.**

Table 1: Pattern of the iterative digit sum of Fibonacci numbers.

Iterative digit sum	Terms
1	1, 25, 49, …
1	2, 26, 50, …
2	3, 27, 51, …
3	4, 28, 52, …
5	5, 29, 53, …
8	6, 30, 54, …
4	7, 31, 55, …
3	8, 32, 56, …
7	9, 33, 57, …
1	10, 34, 58, …
8	11, 35, 59, …
9	12, 36, 60, …
8	13, 37, 61, …
8	14, 38, 62, …
7	15, 39, 63, …
6	16, 40, 64, …
4	17, 41, 65, …
1	18, 42, 66, …
5	19, 43, 67, …
6	20, 44, 68, …
2	21, 45, 69, …
8	22, 46, 70, …
1	23, 47, 71, …
9	24, 48, 72, …

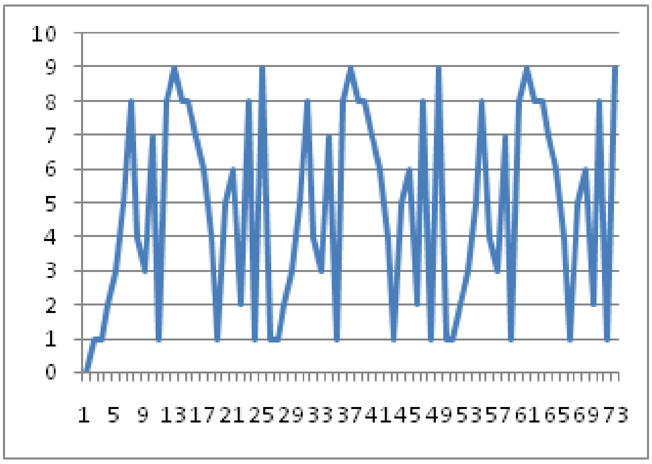

Figure 1: The iterative digit sum of Fibonacci numbers.

Also even Fibonacci numbers map strictly to each other and the same applies to odd numbers. The first two digits of the corresponding recursive numbers are almost the same. It should be noted that some of the results of the iterative digit sum of Fibonacci sequence are available in scientific literature.

Digit and iterative sum of digit of some selected Fibonacci identities

The d'Ocagne's identity is given as:

$$F_{2n} = F_{n+1}^2 - F_{n-1}^2 = F_n(F_{n+1} + F_{n-1}) = F_n L_n \quad (1)$$

We are to show that the sum of the digit of both the L.H.S and R.H.S. of equation 2 are the same. The equation 2 is obtained from 1. The F represents Fibonacci numbers and L represents Lucas numbers.

$$F_{2n} = F_n L_n \quad (2)$$

The digit sum and iterative digit sum of both sides of the equation gave different results. The same was observed when Cassini's identity was considered. The Cassini identity is given by;

$$F_{3n+1} = F_{n+1}^3 + 3F_{n+1}F_n^2 - F_n^3 \quad (3)$$

The reasons of as why the RHS and LHS of the Fibonacci identities are different are due to the algebraic operation differences or heterogeneity of algebraic operators.

Digit and iterative sum of digit of squared Fibonacci Numbers

The sequence of squared Fibonacci numbers is given by;

0, 1, 1, 4, 9, 25, 64, ... (L)

The sequence of integers generated by the digit sum of squared Fibonacci numbers are almost the same sequence generated from the sum of digit of squared positive integers with few terms missing. The iterative digit sum of squared Fibonacci numbers produced numbers that are recursive, a pattern of numbers that are repeated after every 24th consecutive terms. This is shown in **figure 2**. This can also be used in predicting the iterative digit sum of large squared Fibonacci numbers. This is shown in **Table 2**.

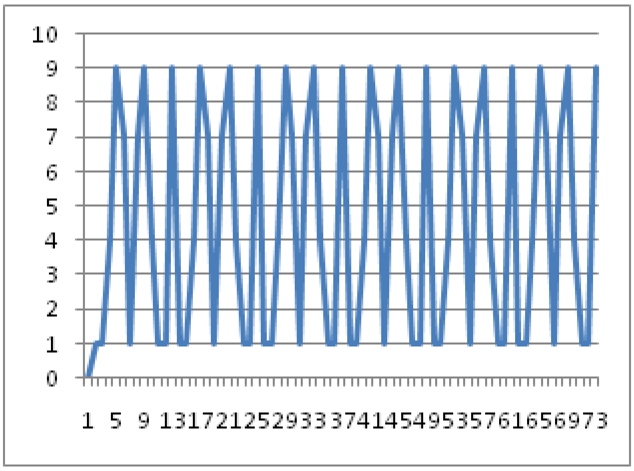

Figure 2: The iterative digit sum of squared Fibonacci numbers.

Table 2: Pattern of the iterative digit sum of squared Fibonacci numbers.

Iterative Digit Sum	Terms
1	1, 25, 49, …
1	2, 26, 50, …
4	3, 27, 51, …
9	4, 28, 52, …
7	5, 29, 53, …
1	6, 30, 54, …
7	7, 31, 55, …
9	8, 32, 56, …
4	9, 33, 57, …
1	10, 34, 58, …
1	11, 35, 59, …
9	12, 36, 60, …
1	13, 37, 61, …
1	14, 38, 62, …
4	15, 39, 63, …
9	16, 40, 64, …
7	17, 41, 65, …
1	18, 42, 66, …
7	19, 43, 67, …
9	20, 44, 68, …
4	21, 45, 69, …
1	22, 46, 70, …
1	23, 47, 71, …
9	24, 48, 72, …

Digit and iterative sum of digit of cubed Fibonacci Numbers

The sequence of cubed Fibonacci numbers is given by;

$$0, 1, 1, 8, 27, 125, 512, \ldots \qquad \text{(M)}$$

The number sequences generated by the digit sum of cubed Fibonacci numbers are almost the same sequence generated from the sum of digit of cubed positive integers with few terms missing. The numbers increase with the cubed Fibonacci numbers. The iterative digit sum of cubed Fibonacci numbers produced numbers that are recursive, a pattern of numbers that are repeated after every 16th consecutive terms. This is shown in **figure 3**.This can also be used in predicting the iterative digit sum of large cubed Fibonacci numbers. This is shown in **Table 3**.

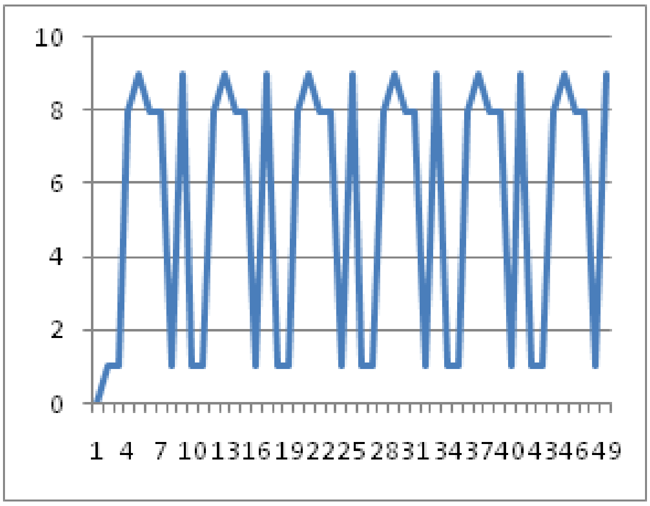

Figure 3: The iterative digit sum of cubed Fibonacci numbers.

Table 3: Pattern of the iterative digit sum of cubed Fibonacci numbers.

Iterative digit sum	Term
1	1, 17, 33, …
1	2, 18, 34, …
8	3, 19, 35, …
9	4, 20, 36, …
8	5, 21, 37, …
8	6, 22, 38, …
1	7, 23, 39, …
9	8, 24, 40, …
1	9, 25, 41, …
1	10, 26, 42, …
8	11, 27, 43, …
9	12, 28, 44, …
8	13, 29, 45, …
8	14, 30, 46, …
1	15, 31, 47, …
9	16, 32, 48, …

Digit and iterative sum of digit of Fibonacci Numbers raised to power 4

The sequence of Fibonacci numbers raised to the power of 4 is given by;

$$0, 1, 1, 16, 81, 625, 4096, \ldots \qquad \text{(N)}$$

The number sequences generated by the digit sum of Fibonacci numbers raised to power of 4 are almost the same sequence generated from the sum of digit of positive integers raised to the power of 4, with moderate terms missing. The iterative digit sum of Fibonacci numbers raised to the power of 4 produced numbers that are recursive, a pattern of numbers that are repeated after every 12^{th} consecutive terms. This is shown in **figure 4**.This can also be used in predicting the iterative digit sum of large Fibonacci numbers raised to the power of 4. This is shown in **Table 4**.

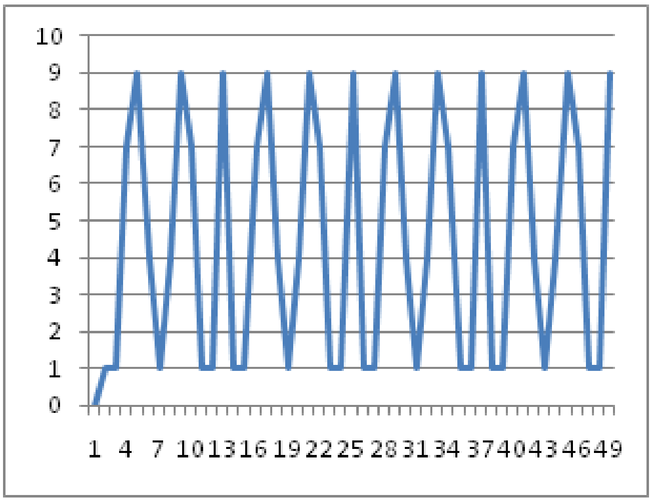

Figure 4: The iterative digit sum of Fibonacci numbers raised to the power of 4.

Table 4: Pattern of the iterative digit sum of Fibonacci numbers raised to the power of 4.

Iterative digit sum	Term
1	1, 13, 25, …
1	2, 14, 26, …
7	3, 15, 27, …
9	4, 16, 28, …
4	5, 17, 29, …
1	6, 18, 30, …
4	7, 19, 31, …
9	8, 20, 32, …
7	9, 21, 33, …
1	10, 22, 34, …
1	11, 23, 35, …
9	12, 24, 36, …

Digit and iterative sum of digit of Fibonacci Numbers raised to power 5

The sequence of Fibonacci numbers raised to the power of 5 is given by;

$$0, 1, 1, 32, 243, 3125, 32768, ... \qquad (O)$$

The number sequence generated by the digit sum of Fibonacci numbers raised to power of 5 are almost the same sequence generated from the sum of digit of positive integers raised to the power of 5, with moderate terms missing. The iterative digit sum of Fibonacci numbers raised to the power of 5 produced numbers that are recursive, a pattern of numbers that are repeated after every 24^{th} consecutive terms. This is shown in **figure 5**.This can also be used in predicting the iterative digit sum of large Fibonacci numbers raised to the power of 5. This is shown in **Table 5**.

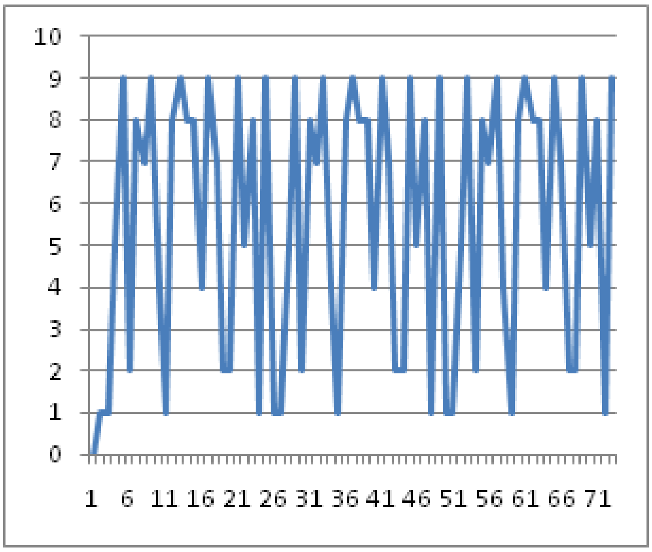

Figure 5: The iterative digit sum of Fibonacci numbers raised to the power of 5.

Table 5: Pattern of the iterative digit sum of Fibonacci numbers raised to the power of 5.

Iterative Digit Sum	Terms
1	1, 25, 49, …
1	2, 26, 50, …
5	3, 27, 51, …
9	4, 28, 52, …
2	5, 29, 53, …
8	6, 30, 54, …
7	7, 31, 55, …
9	8, 32, 56, …
4	9, 33, 57, …
1	10, 34, 58, …
8	11, 35, 59, …
9	12, 36, 60, …
8	13, 37, 61, …
8	14, 38, 62, …
4	15, 39, 63, …
9	16, 40, 64, …
7	17, 41, 65, …
2	18, 42, 66, …
2	19, 43, 67, …
9	20, 44, 68, …
5	21, 45, 69, …
8	22, 46, 70, …
1	23, 47, 71, …
9	24, 48, 72, …

CONCLUSION

It was observed that as the power of the Fibonacci numbers increases from 1 to the power of 5, the number of terms obtained from the digit sum decreases when compared with their respective positive integers. Palindromic numbers was a subset of numbers obtained from the sum of digit of Fibonacci numbers. The results of the analysis of the two Fibonacci identities indicated that digit sum may not be applicable to Fibonacci identities because of the heterogeneity of algebraic operations. Iterative digit sum have added to the scientific literature as one of the various means of dividing integer sequences into distinct classes. In this case, the notion of the iterative digit sum was used to divide or partition the Fibonacci numbers into 9 distinct classes. Different recursive patterns of the iterative digit sum were obtained for the different powers of the Fibonacci numbers considered. This is an indication that the sequences are unique. The result of the digit sum of the squared and cubed Fibonacci are almost the same with the results of [7] and [8]. Prediction of the iterative digit sum of the powers of Fibonacci can be obtained by simple computation as illustrated in the paper. Given any power of Fibonacci number, the iterative digit sum can be computed based on the patterns as shown in this paper. The results of this research can be compared with [9] and [10], which are also results available on the digit and iterative digit sum of sequences of numbers.

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