Analysis of Stochastic Model of Market Sales Record

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***Abstract*— Firms in Nigeria and other parts of the world are constantly faced with the problems of production of goods and services or cost of production visa vise customers' satisfaction need, prices attached to these goods, services and profits. The challenge of trying to meet with the customers' needs in terms of quality and quantity within best price practices (marginal profit) remains a vital part of decision making in all producing firms (manufacturing, service rendering companies, etc.). This research is born out of a concern for such challenges. The research addresses the issues of customer satisfaction needs based on four different brands of foam (mattresses) from a foam producing company in Lagos, Nigeria by analyzing the sales record of the different brands of foam using a simple transition probability matrix of a Markov chain. The analysis of a three - months sales record of the four brands of foams is studied using a stochastic model called Markov chain (M.C.). The sales record covered the three premier regions; North, West and East. The results of the analysis show that customer's satisfaction need can be tied to so many factors: economy, quality of foam, family size, status and purpose of purchase. Analysis of the transition probabilities shows that brands B, which is a premium class product, is the most preferred brand by the customers, followed by brand C, which is a middle-class product. The foam producing company can use this analysis to slow down the production of brands A and D and increase the production of brands B and C. Instead of scraping out Brands A and D completely from production, the company should adopt new advertising strategies to promote brands A and D to improve on their sales.**

***Keywords*—** *Sales analysis, stochastic model, Markov chain, Absorbing state, Regular Markov chain, Empirical data.*

1. Introduction

Classical statistical analysis has been used extensively to analyze sales data from various producing and service rendering firms [7, 9-11, 14, 15, 18]. Some of the reasons for using statistical models to analyze sales data of particular products is for forecasting, determination of order of preferences, brand improvement and flexible manufacturing to avoid waste of limited resources. Up till now, many businesses are faced with the challenges of predicting customers' purchasing behavior using their historical transaction data.

The advent of strong computational capabilities of machines and advanced mathematical techniques has propelled experts and technocrats to venture into an era where forecasting charts can be used for future business plans and events. Over the years, the traditional methods of prediction and forecasting have been heavily replaced by faster and advanced methods that can handle huge data that cannot be handled by the traditional methods. Uneven business cycles or seasonal variations such as weather, holiday, government policies and natural disasters have increased the uncertainties involved in forecasting business transactions and to accurately predict customers' behavior. Profitability and existence of business venture depends on effective analysis of sales and interpretation of the results to aid crucial decision making such as production, recruitment, marketing, diversification and so on. Insight obtained are birthed by careful analysis of data and implementation of decision [9, 13, 15, 19].
Manufacturing and production companies are most often searching for avenue for will create a feedback from their customers. The results from the feedback and historical data can help build a robust predictive model for a satisfactory prediction of customer lifetime value behavior while optimizing the available scarce resources. The choice of the predictive models is often the one of the major challenges encountered in this context. Several models in the literature used for such purposes are not easy to implement in practice. Organizations usually prefer the sophisticated models to deal with the huge amount of data generated by them. When pattern is the aim, machine learning or data mining

IJSCM, ISSN: 2050-7399 (Online), 2051-3771 (Print)

tools are the best option. However, simple methods are easier to compute and implement and serve for small and medium scale firms that may not afford to use sophisticated models.

In [7, 14], statistical analysis was carried out on a dataset that describes some variables associated with the number of car sales with the sole purpose of determining the customers' preferences.

In [18], Hedonic models were used to predict the selling prices of properties but the model which is similar to multiple regression do not take into cognizance, the random nature of time and price. The model is deterministic and slight changes in price can render the models ineffective.

On the other hand, Markov Chain also known as stochastic matrix, which satisfies the Markov property, has been used for prediction of future outcomes of processes based solely on the present state just as the complete history of a process can be known by conditioning on the present state of the process [9, 15, 19, 22, 25]. Markov forecasting is a mathematical model used as a predictive model in which the probability theory of Markov chain is applied to study the effect of change time on the economic or business variables. Often, price is being studied and the method has been used by many authors. See [12, 16, 24] and the references therein.

Markov chains have found many applications as statistical models of real-world processes [1-8, 14, 17, 20] such as studying behaviors of queues at bus stations, airports, banks, sales data represented by market share, etc.

Zhang et al. [23] employed the discrimination and location method-supply chain-root Cause (SC-RC) as well as Markov prediction method to study the problem based on a series of factors such as the supply and demand in solving the frequent occurring fruit and vegetable agricultural products problems with the increasing varieties of unsaleable goods. The identification of crucial causes of unsaleable fruit and vegetable agricultural products and their risk prediction were studied.

Zhou et al. [25] used real data from electricity market investigation to construct a Markov chain model with a grey transfer probability matrix on residential energy consumption items such as heating, cooking, cooling, etc. This is to help the service providers to be clear about market distribution of residential energy consumption in the future. Ching et al. [5] carried out a study on the effect of higher-order Markov chain models for analyzing categorical data sequences such as DNA and sales demand to understand users' behavior in accessing information and to predict their behavior in the future. Cheng et al. [4] used three combined data mining models, regression analysis and Markov chain to obtain estimates from customers' past transactions to predict customers' lifetime value which involve an estimation of customer purchasing power, likelihood of future spending and the expected profit associated with each behavior of the customer. The Markov chain model was mainly used to model the transition probabilities of behavior change of customers. See also [10, 21]. Also, Li et al. [12] applied Markov chain theory to the actual market share analysis of municipal automobiles sales market with empirical data.

Performing sales trend analysis gives much needed understanding into the inner workings of business. Business owners use their data to make informed decisions for instance, when to raise or lower prices of their products, when to reduce the production of a product due to demand and when to update on the quality or taste of products. In trend analysis of sales data, the aim is often to determine opportunities, strengths, weaknesses and threats of a particular product tracked by the in-depth synthesis of the sales. Consequently, timely decisions can be made. Hence, marketing, advertising and sales promotions hugely depends on the trends of sales.

This paper presents a simple forecasting model that can be used to predict the operational/sales data on a weekly basis using Markov chain (M.C.). A three - month sales record of four brands of foams (mattresses) from a foam producing company in Western Nigeria is studied using a stochastic model called Markov chain. The sales record covered the three premier regions; North, West and East. This is because they are the most populated regions in Nigeria. A four states M.C. is constructed for each period (January, May and September) to factor the seasonal changes that might also affect sales (rainy, dry and festive seasons). The states are a finite set of relation states.

The objective of this study is to analyze the sales records of the products using a simple Markov chain. In addition, the study attempts to determine the most viable product and its prospect in order to make corresponding sales management strategy advise the company for effective planning for future production. The customers' choice of brand is assumed to be directly proportional to the sales of the brand. This is based on the percentage of sales of each brand or market share. Customers are attracted to different brands for various reasons (customers' purchasing power, quality of product, texture of product (that is hard, soft or medium), family size, social status, etc.).

2. Methodology

The following transition matrices were determined from the sales record of four products of foam which represents the market share:

1. High premium class product brand is denoted by A. This product, as the name implies is the most durable product produced with high quality materials and the most expensive.
2. Premium class product brand is denoted by B. This is of course a lower version of brand A and less expensive than brand A.
3. Middle class product brand is denoted by C. This is of a lower quality compared to brand B and
4. Low-end class product brand is denoted by D. This is the least in terms of quality and durability, hence, the cheapest.

$$P_1 = \begin{bmatrix} & A & B & C & D \\ A & 0.06 & 0.03 & 0.56 & 0.35 \\ B & 0.53 & 0 & 0.47 & 0 \\ C & 0.32 & 0.2 & 0.31 & 0.17 \\ D & 0.18 & 0.32 & 0.3 & 0.2 \end{bmatrix}$$

Fig 1 Markov Chain for January

$$P_2 = \begin{bmatrix} & A & B & C & D \\ A & 0 & 0.47 & 0.29 & 0.24 \\ B & 0 & 0.35 & 0.38 & 0.27 \\ C & 0 & 0.46 & 0.29 & 0.25 \\ D & 0.22 & 0.28 & 0.20 & 0.30 \end{bmatrix}$$

Fig 2 Markov Chain for May

$$P_3 = \begin{bmatrix} & A & B & C & D \\ A & 0.47 & 0 & 0 & 0.53 \\ B & 0 & 1 & 0 & 0 \\ C & 0.7 & 0 & 0 & 0.3 \\ D & 0.23 & 0.34 & 0.22 & 0.21 \end{bmatrix}$$

Fig 3 Markov Chain for September

The three Markov chains (M.C.) P_1, P_2 and P_3 show the brands of the foams and their sales transition from state to state in three months (January, May and September). The entries of these Markov chains represent the market shares of each brand of foam and their transition from one state to another, which also shows how customers switch from one brand to another.

Remark: The following analysis is carried out to show that P_1, P_2 and P_3 satisfied the properties of a M.C.:

- The matrices P_1, $P_2$and P_3 are right stochastic matrices since all the rows of P_1, P_2 and P_3 sums to one.
- The four state space is finite, hence, the transition probability distribution can be represented by a matrix, called the transition matrix with (i, j) th element of P_1, P_2 and P_3 equal to

$$P_{ij} = \Pr(X_{n+1} = j \mid X_n = i) \quad (1)$$

The following analysis gives the interpretation of the Markov chains P_1, P_2 and P_3 for the first month sales:

From P_1(Fig 1) [Occurred between January and April]

1). Someone using brand A will most likely stay with brand A with 6% probability and someone not using brand A will switch to brand A with 53%, 32% and 18% respectively.
2). The probability of sticking to brand B is 0% while the probability of switching from other brands to brand B is 2%, 32% and 3% respectively.
3). A customer using brand C will most likely stay with brand C with 31% probability and someone not using brand C will switch to brand C with 56%, 47% and 3% respectively.
4). The probability that a customer who is using brand D will continue using brand D is 2%, while the probability of someone not using brand D will switch to brand D is 35%, 0%, and 17% respectively.

From P_2 (Fig 2) [Occurred between May and August]

1). The probability that someone is using brand A is 0%, while the probability that someone not using brand A will switch to brand A are 0%, 0% and 22% respectively.
2). The probability of using and sticking to brand B is 35% while the probability of switching from other brands to brand B are 47%, 46% and 28% respectively.
3). A customer using brand C will most likely stay with brand C with 29% probability and someone not using brand C will switch to brand C with 29%, 38% and 20% respectively.
4). The probability that a customer who is using brand D will continue using brand D is 30%, while the probability that someone not using brand D will switch to brand D is 24%, 27%, and 30% respectively.

From P_3 (Fig 3) [Occurred between September and December]

1). Customers using brand A will most likely stay with brand A with 47% probability and someone not using brand A will switch to brand A with probabilities 0%, 7% and 23% respectively.
2). The probability of sticking to brand B is 100% while the probability of switching from other brands to brand B is 34%.
3). The probability of using brand C and staying with brand C is 0%, while someone not using brand C will switch to brand C with only 22% probability.
4). The probability that a customer who is using brand D will continue using brand D is 21%, while the probability that someone not using brand D will switch to brand D is 53% and 3% respectively.

3. Result and Discussion

Figures 4-6 represent the n-step transition probability matrix of P_1, P_2 and P_3. The n-step transition probability matrix of a M.C. shows the long run behaviour of the M.C. after some steps. If the M.C. attains stationary probabilities (steady state) after a long period of time (n-steps) which do not change no matter the initial state of the chain, then the M.C. is said to be ergodic. MATLAB is used to determine the stationary probabilities.

$$P_1 = \begin{bmatrix} 0.06 & 0.03 & 0.56 & 0.35 \\ 0.53 & 0 & 0.47 & 0 \\ 0.32 & 0.2 & 0.31 & 0.17 \\ 0.18 & 0.32 & 0.3 & 0.2 \end{bmatrix}$$

$$P_1^2 = \begin{bmatrix} 0.2617 & 0.2258 & 0.3263 & 0.1862 \\ 0.1822 & 0.1099 & 0.4425 & 0.2654 \\ 0.2550 & 0.1260 & 0.4203 & 0.1987 \\ 0.3124 & 0.1294 & 0.4042 & 0.1540 \end{bmatrix}$$

$$P_1^3 = \begin{bmatrix} 0.273308 & 0.132695 & 0.409691 & 0.184306 \\ 0.258551 & 0.178894 & 0.370480 & 0.192075 \\ 0.252342 & 0.155294 & 0.391923 & 0.200441 \\ 0.244390 & 0.139492 & 0.407264 & 0.208854 \end{bmatrix}$$

$$P_1^4 = \begin{bmatrix} 0.25100303 & 0.14911536 & 0.39771514 & 0.20216647 \\ 0.26345398 & 0.14331653 & 0.40134004 & 0.19188945 \\ 0.25894108 & 0.15009598 & 0.39592813 & 0.19503481 \\ 0.25651236 & 0.15561778 & 0.39132768 & 0.19654218 \end{bmatrix}$$

$$P_1^5 = \begin{bmatrix} 0.2577501320 & 0.1517663893 & 0.3945875504 & 0.1958959283 \\ 0.2547339135 & 0.1495762514 & 0.3968752453 & 0.1988145898 \\ 0.2568906016 & 0.1493649976 & 0.3968002787 & 0.1969441221 \\ 0.2584706150 & 0.1488544044 & 0.3970615130 & 0.1956134676 \end{bmatrix}$$

$$P_1^6 = \begin{bmatrix} 0.257430477471 & 0.149336711096 & 0.396761196005 & 0.196471615428 \\ 0.257346152712 & 0.150637735201 & 0.395627532701 & 0.196388579386 \\ 0.257002915986 & 0.150088892860 & 0.396151608795 & 0.196756582359 \\ 0.256671179560 & 0.149762730682 & 0.396478223778 & 0.197087865980 \end{bmatrix}$$

Fig 4 6-step transition probability matrix of P_1

$$P_2 = \begin{bmatrix} 0 & 0.47 & 0.29 & 0.24 \\ 0 & 0.35 & 0.38 & 0.27 \\ 0 & 0.46 & 0.29 & 0.25 \\ 0.22 & 0.28 & 0.20 & 0.30 \end{bmatrix}$$

$$P_2^2 = \begin{bmatrix} 0.0528 & 0.3651 & 0.3107 & 0.2714 \\ 0.0594 & 0.3729 & 0.2972 & 0.2705 \\ 0.0550 & 0.3644 & 0.3089 & 0.2717 \\ 0.0660 & 0.3774 & 0.2882 & 0.2684 \end{bmatrix}$$

$$P_2^3 = \begin{bmatrix} 0.059708 & 0.371515 & 0.298433 & 0.270344 \\ 0.059510 & 0.370885 & 0.299216 & 0.270389 \\ 0.059774 & 0.371560 & 0.298343 & 0.270323 \\ 0.059048 & 0.370834 & 0.299810 & 0.270308 \end{bmatrix}$$

$$P_2^4 = \begin{bmatrix} 0.05947568 & 0.37106851 & 0.29910539 & 0.27035042 \\ 0.05948558 & 0.37112773 & 0.29904464 & 0.27034205 \\ 0.05947106 & 0.37106800 & 0.29911133 & 0.27034961 \\ 0.05946776 & 0.37114330 & 0.29904734 & 0.27034160 \end{bmatrix}$$

$$P_2^6 = \begin{bmatrix} 0.05947509568 & 0.371110270771 & 0.299069300963 & 0.270344718698 \\ 0.059475856286 & 0.371110672261 & 0.299068799168 & 0.270344672285 \\ 0.059475708578 & 0.371110252672 & 0.299069315381 & 0.270344723369 \\ 0.059475959048 & 0.371110786282 & 0.299068594238 & 0.270344660432 \end{bmatrix}$$

$$P_2^7 = \begin{bmatrix} 0.05947583811356 & 0.37111057794523 & 0.29906889968657 & 0.27034468425464 \\ 0.05947582790270 & 0.37111054360285 & 0.29906893999784 & 0.27034468849661 \\ 0.05947583914118 & 0.37111057908544 & 0.29906889763727 & 0.27034468413611 \\ 0.05947582529504 & 0.37111053422170 & 0.29906895132650 & 0.27034468915676 \end{bmatrix}$$

$$P_2^8 = \begin{bmatrix} 0.0594758305360208 & 0.3711105516413251 & 0.2990689304321531 & 0.2703446873905010 \\ 0.0594758314692542 & 0.3711105545533237 & 0.2990689269595616 & 0.2703446870178605 \\ 0.0594758305099442 & 0.3711105515475136 & 0.2990689305454397 & 0.2703446873971025 \\ 0.0594758316144872 & 0.3711105554403466 & 0.2990689260558446 & 0.2703446868893216 \end{bmatrix}$$

Fig 5 8-step transition probability matrix of P_2

$$P_3 = \begin{bmatrix} 0.47 & 0 & 0 & 0.53 \\ 0 & 1 & 0 & 0 \\ 0.7 & 0 & 0 & 0.3 \\ 0.23 & 0.34 & 0.22 & 0.21 \end{bmatrix}$$

$$P_3^2 = \begin{bmatrix} 0.3423 & 0.1802 & 0.1166 & 0.3604 \\ 0 & 1 & 0 & 0 \\ 0.398 & 0.102 & 0.066 & 0.434 \\ 0.3104 & 0.4114 & 0.0462 & 0.2320 \end{bmatrix}$$

$$P_3^3 = \begin{bmatrix} 0.325628 & 0.302736 & 0.079288 & 0.292348 \\ 0 & 1 & 0 & 0 \\ 0.33308 & 0.24956 & 0.09548 & 0.32188 \\ 0.231588 & 0.490280 & 0.051040 & 0.227092 \end{bmatrix}$$

$$P_3^4 = \begin{bmatrix} 0.27578680 & 0.40213432 & 0.06431656 & 0.25776232 \\ 0 & 1 & 0 & 0 \\ 0.2974160 & 0.3589992 & 0.0708136 & 0.2727712 \\ 0.19680552 & 0.56749128 & 0.04996024 & 0.18574296 \end{bmatrix}$$

$$P_3^5 = \begin{bmatrix} 0.2339267216 & 0.4897735088 & 0.0567077104 & 0.2195920592 \\ 0 & 1 & 0 & 0 \\ 0.252092416 & 0.451741408 & 0.060009664 & 0.236156512 \\ 0.1701916432 & 0.6306438864 & 0.0408634512 & 0.1583010192 \end{bmatrix}$$

$$P_3^6 = \begin{bmatrix} 0.200147130048 & 0.564434808928 & 0.048310253024 & 0.187107808000 \\ 0 & 1 & 0 & 0 \\ 0.21480619808 & 0.53203462208 & 0.05195443264 & 0.20120474720 \\ 0.145003722560 & 0.684466232928 & 0.034826224224 & 0.135703820288 \end{bmatrix}$$

$$P_3^7 = \begin{bmatrix} 0.17092112407936 & 0.62805146364800 & 0.04116371776000 & 0.15986369451264 \\ 0 & 1 & 0 & 0 \\ 0.1836041078016 & 0.6004442361280 & 0.0442650443840 & 0.1716866116864 \\ 0.12374198522624 & 0.73060553182592 & 0.02985484046336 & 0.11579764248448 \end{bmatrix}$$

$$P_3^8 = \begin{bmatrix} 0.1459161804872064 & 0.6824051197822976 & 0.0351700127927808 & 0.1365086869377152 \\ 0 & 1 & 0 & 0 \\ 0.156767382423424 & 0.658817684101376 & 0.037771054571008 & 0.146643878904192 \\ 0.1056905791521152 & 0.7699767302706432 & 0.0254754813465856 & 0.0988572092306560 \end{bmatrix}$$

Fig 6 8-step transition probability matrix of P_3

The distribution over states can be written as a stochastic row vector x with the relation

$$x^{(n+1)} = x^{(n)} p \; .$$

Therefore, if the system is in state $x^{(n)}$ at time n, then in four steps, that is, at time $n+4$, the distribution will be

$$x^{(n+4)} = x^{(n+3)} p(x^{(n+2)} p) p = (x^{(n+1)} p) p^2$$
$$= x^{(n+1)} p^3 = (x^{(n)} p) p^3$$
$$= x^{(n)} p^4$$

In particular, if at time n the system P_1 is in state 3 (brand C), then at time $n+5$, the distribution is

$$x^{(n+3)} = [0\ 0\ 1\ 0] \begin{bmatrix} 0.06 & 0.03 & 0.56 & 0.35 \\ 0.53 & 0 & 0.47 & 0 \\ 0.32 & 0.2 & 0.31 & 0.17 \\ 0.18 & 0.32 & 0.3 & 0.2 \end{bmatrix}^5$$

$$x^{(n+3)} = [0\ 0\ 1\ 0] \begin{bmatrix} 0.2576 & 0.1518 & 0.3946 & 0.1959 \\ 0.2547 & 0.1496 & 0.3969 & 0.1988 \\ 0.2569 & 0.1494 & 0.3968 & 0.1969 \\ 0.2585 & 0.1489 & 0.3971 & 0.1956 \end{bmatrix}$$

$$= [0.2569\ 0.1494\ 0.3968\ 0.1969]$$

Indicating that brand A has 25.7% of the market share, 14.9% for brand B, 39.7% for brand C and 19.7% for brand D. By taking the limit as $n \to \infty$, we obtain;

$$\lim_{n\to\infty} P_1^n = \begin{bmatrix} 0.257 & 0.150 & 0.400 & 0.200 \\ 0.257 & 0.150 & 0.400 & 0.200 \\ 0.257 & 0.150 & 0.400 & 0.200 \\ 0.257 & 0.150 & 0.400 & 0.200 \end{bmatrix}$$

Assume that the initial state matrix is given by

$$\begin{matrix} & A & B & C & D \end{matrix}$$
$$S_0 = [0.30\ 0.17\ 0.32\ 0.21]$$

Which means that at the start (beginning of the year), brands A, B, C and D control 30%, 17%, 32% and 21% of the market share respectively.

Note: 'step' here, denotes 'Weeks'.

After the second step transition matrices P_1^2, P_2^2 and P_3^2 (**Fig 4 - 6**), it is observed that all states in each of these Markov chains have the probability of been occupied. That is, all the brands have the probability of being purchased by customers except for P_3^2 where the customers stayed with brand B throughout the period. At that same period, other brands recorded zero sales (see, P_1^2, P_2^2 and P_3^2 in **Fig 4 - 6**).

The first transition probability matrix P_1 attained its stationary transition matrix in six steps. This shows that after a long period of time (say six weeks), brands A, B, C and D will have 26%, 15%, 40% and 20% of market share respectively. A comparison with the initial state distribution matrix S_0, shows that after a long period of time, brand C will take the highest share of the market.

There are no significant changes in the market shares of brands A and D when compared with the initial state distribution. Although, a slight improvement of 4% and 2% in the market shares for brands A and D respectively. On the other hand, brand B experienced a slight decline in the market share from 17% to 15%.

The second transition probability matrix P_2 represents the market share of the four brands for the second four months of the year. The results in P_2^8 show that P_2 attained its stationary probability after eight (8) steps. This indicates that the market was a bit slow in the months of June when compared with P_1 which attained its stationary probabilities in just six (6) steps. Here, a sharp drop from 33% to 6% in the market share was observed for brand A. Brands B and D experienced increase in their market shares. Brand B increased from 17% to 37% and brand D increased from 21% to 27%. In the case of brand C, there was a slight drop in the market share from 32% to 30%.

The drop in the market share of brand A can be attributed to some factors such as the decline in the buying power of the customers due to the high price of the brand. This corroborates the theory of demand and price. The outcome can also be due to low level of advertising endeavours as a result of low production necessitated by low demand and high cost of raw materials required to produce the brand. Technical factors can also be a reason.

For P_1 and P_2, the probability of moving from one brand to another is greater than zero. This is because the Markov chains are **regular**. That is, all entries are greater than zero after a long period.

The transition probability matrix P_3 is not a regular Markov chain because even after eight steps, some of the entries were still zero. This implies that the probability of moving from brand B to A after a long period of time did not improve. It could be due to high cost of brand A while customer who purchased brand B are most likely to remain loyal to the brand, no matter the number of steps. (See **Fig 6**). The probability of moving from brand B to brands A, C and D is also zero, an indication that customers are likely to remain with brand B. This is because brand B, though a lower grade of brand A, is a better quality than C and D, and cheaper than brand A.

The transition probability matrix P_3^8 shows that customers prefer brand B to any other brand. The results show that movement from brand A to B is 68%, C to B is 66% and D to B is 77%.

For the probability transition matrix P_3, the limit and the stationary distribution do not exist. This is clearly depicted from P_3^2 to P_3^8 and the formula

$\lim_{n \to \infty} P_3^k = 1\pi$ cannot be applied because P_3 is not a periodic Markov Chain. The symbol π represents the stationary distribution.

At the beginning of the last trimester of the year P_3, there was no indication that brand B will eventually have the highest market share. This can be seen from the following analysis of P_3 in **Fig 4.**

1). Movement from brand A to B = 0.
2). Movement from brand A to B = 100%.
3). Movement from brand A to B = 0.
4). Movement from brand A to B = 34%.

The M.C. P_3 in the last month of the year is an **absorbing** M.C. since the entry BB, (i.e., movement from brand B to B) is an absorbing state, that is $P_{BB} = 1, P_{ij} = 0$ while the M.C. P_1 and P_2 are ergodic. Ergodicity implies that all the states or entries (brands) have the probability of been occupied (purchased).

4. Conclusion

It was observed from the transition probability matrices P_1^2, P_2^2 and P_3^2, that all states in each of these Markov chains have the probability of been occupied. That is, all the brands have the probability of being purchased by customers but at different rates. The transition probability matrix P_3^8 shows that customers prefer brand B to any other brand. The results indicate that movement from brand A to B is 68%, C to B is 66% and D to B is 77%. The results show that brand B, which is a premium class product, is the most preferred brand by the customers, followed by brand C, which is a middle-class product. The foam producing company can use this analysis to slow down the production of brands A and D and increase the production of brands B and C instead of scraping brands A and D from production. More advertising and promotion of brands A and D are suggested as a strategy to improve on their sales.

Conflict of Interest: The authors declare that there is no conflict of interest.

Acknowledgments

The authors would like to thank Covenant University for the support.

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